Magnetic Fields in the Heavens

A national physics competition problem set on stellar magnetic fields and the dynamo effect.

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Total: 49 marks · Coffee (8) · Uniform gas (9) · Stars (3) · Dynamos (29)

There was once talk of using the magnetic compass to navigate space. Magnetic field lines permeate space and are responsible for a plethora of phenomena, from pulsars to the auroras. We'll study and model the origin of the magnetic fields of celestial bodies in the following problems.

The magnetic field is easiest to model for stars due to homogenous composition. The only true insight needed to solve the following problems is that "magnetic fields arise from moving charges".

§1 Coffee

The magnetic field of stars arises out of convection currents. Convection is a complex physical phenomenon.

First, draw a neat diagram of convection current at steady state for coffee put to boil in a perfectly cylindrical container, heated by a point source at the very center. The pH of coffee is typically 5. Find out the expression for magnetic field generated by the convection of Hydrogen ions inside the container. [1 mark]

Bonus. Interestingly, you don't have to worry about any convection loops carrying of varying sizes which carry the H^+ current. Only large loops adjacent to the surface of the container and liquid will be present in steady state. Explain in 100 words why. [5 marks]

Now, you most probably arrived at an answer using circular or square convection loops. What if the loop is elliptical? Or irregular? How would the expression change? Draw the detailed magnetic field for both inside and outside the container for an irregular loop. [1 mark]

That was easy, right?! Now we can think about more complicated models.

What would convection look like if the coffee pot was heated uniformly at the base? Draw a neat diagram with a description not exceeding 100 words. [1 mark]

§2 Uniform Gas

Now, let the temperature at the bottom of the container be T. Humor me and consider the temperature at the top of the container as $0\,\mathrm{K}$ and the coffee is replaced by an ideal gas. The temperature varies with height as a linear function. The walls are adiabatic, the system is in a steady state. Under what conditions will convection not occur? Derive an expression. [2 marks]

Hint. I know this sounds painful so let's break it down. A packet ascends if its density at every point is less than the surrounding medium. Write out the pressure expression for an ideal gas and a fluid under gravity. You clearly want an inequality… what can you substitute in these expressions to get one? Now use this insight to math it out!

How about convection in a sphere of radius R held together by its own gravity heated by a tiny point source at its center at temperature T? The temperature varies as $T(r)$, and the adiabatic constant can be taken as γ. [2 marks]

Bonus. Can you generalize this expression in terms of arbitrary T and P? [5 marks]

§3 Stars

The magnetic field of stars arises out of convection currents. It has been noticed that main-sequence stars with over 10 solar masses have very low surface magnetic fields, while red dwarves have strong superficial magnetic fields. Can you use the expression developed in the previous model to qualitatively model the inner structure of the star? You should write about the variance of temperature with depth and draw a diagram showing the extent of the convection zones for both types of stars. [3 marks]

§4 Dynamos

Now finally, to the main event! The dynamo effect. We'll be studying this very intriguing piece of astrophysics using a very simple model. In what follows, do not think of the fluid I'm describing as a material, treat it as a collection of points assigned a parameter called charge. There is no interparticle interaction (they can slide over each other, do not consider collisions) and the points are not allowed to move outside the periphery of the ring or disc mentioned.

Consider two concentric rings of radius a and b of fluid with charge density λ, which start rotating at $t = 0$ with ω.

Describe the magnetic field created by this system and describe the force acting on the fluid particles of both rings within 100 words. [4 MARKS]

Now, consider a disc of radius R. Fluid with total charge Q and total mass M. It starts rotating at $t = 0$ with ω. Now, describe the evolution of the magnetic field in this considerably more complex system. To help with the analysis, I'll break this down into subparts.

(a) Describe the motion of the particles. At what times is the radial force zero? At what particular radius does this happen? [4 MARKS]

(b) Describe the new field that arises. Describe its magnitude as a function of radius and draw the shape and direction of its field lines. [4 MARKS]

(c) Now describe the effect of this new field onto particle motion. [4 MARKS]

A model displays the dynamo effect if there's an interconversion between mechanical and magnetic energies. Describe the energy interconversion in this model within 200 words. [4 MARKS]

BONUS. Find the time dependence of magnetic and mechanical energies. [5 MARKS]

For the finale, describe the flow created in a spinning sphere of ideal fluid qualitatively. How would things change if the sphere contains a point heating source at the center? There is no restriction on words. This final part is solely to test your physical insight, your intuition.

It will be very helpful to you if you think in terms of combining all the models developed. Can you draw a diagram of what the large-scale field (i.e., viewed from a distance so that all the internal complications are not visible) of this final model would look like? Do you see any resemblance in all the fields you've drawn so far? Yes, it's amazing! [4 MARKS]

And there you have it. The origin of the magnetic field of stars: convective zones and the accompanying dynamo effect. This applies to most planets too! Some planets like Mercury have small fields due to polarized local depositions of magnetic materials while some like Uranus have very weird wobbly fields that are subjects of active research. Remember that while we've used very simple, idealistic models, a real star's field is quite similar to the one we got for our final model :)