The Mathematics of Change

Dhruv Gupta

Principal Investigator | Founder & Chief Scientific Officer ARCNet

AI and Robotics Tech Park (ARTPARK), Indian Institute of Science

First Principles

Is climate change important?

Concern One

*Is climate change **objectively** important?*

Evidence

Moral importance – climate change carries moral importance.

If I don't act on climate change right now, my friend in Delhi will live 5 years less.

Is this enough?

Moral importance – climate change carries moral importance.

If I don't act on climate change right now, my friend in Delhi will live 5 years less.

Is this enough to make you drop a career in AI and join a climate startup?

Is climate change objectively important, for you?

Evidence

Let's ask – is AI important?

It has a large market of \$244B $\rightarrow$ demand.
A large number of people (7–8M) are working in this field.

The bottom line is opportunity.

Evidence

Climate market – \$851B in 2024 and expected to grow to about \$4T in 2034 (compliance market).

Currently employing about 3M people.

Climate market provides tremendous opportunity.

Climate change is, objectively, important.

Evidence

Climate market provides opportunity to be employed.

Climate market provides opportunity to be paid well.

Climate market provides opportunity to work towards a moral cause.

Evidence

Climate change **is** *objectively important, for you.*
Importance is defined in terms of Opportunity.

Agenda

Concern 2

*What **in** climate change is important?*

Concern 2

The answer to this is different for different people.

Concern 2

For a scientist – there's an opportunity in answering

"What is the outcome of climate change?"

For a policymaker/economist – there's an opportunity in answering

"What is the monetary equivalent of a unit effort in preventing climate change?"

For an entrepreneur – there's an opportunity in answering

"Who's buying what to prevent climate change?"

Concern 2

For an engineer – there's an opportunity in answering

*"How to **build** something that the entrepreneur **can sell with profit margin** under **terms set by** the policymaker that **prevents the bad outcomes** as defined by the scientist?"*

Objective, constraints and goal

Science is bedrock of these motivations

What is climate change and what is its outcome?

Opportunity $\rightarrow$ *theory of climate and models of outcome.*

What do we know about the outcome of climate change?

Apparent Climate DOFs and Compact State Equation

$$x(t) \equiv \begin{bmatrix} T(t) \\ C_a(t) \\ E(t) \end{bmatrix}, \qquad E \approx C_T\, T \text{ (effective heat capacity approximation).}$$

Apparent Climate DOFs and Compact State Equation

$$x(t) \equiv \begin{bmatrix} T(t) \\ C_a(t) \\ E(t) \end{bmatrix}$$

$$\frac{dx}{dt} = f_{\mathrm{free}}\big(x, t;\; F_{\mathrm{solar}}, F_{\mathrm{GHG}}, F_{\mathrm{volc}}, F_{\mathrm{aer}}\big) + f_{\mathrm{bio}}\big(x;\; B(t)\big) + \xi(t). \tag{C0}$$

Energy Balance with Explicit Biosphere Dependence

$$\frac{dE}{dt} = N(t) = \underbrace{(1 - \alpha(B))\,\frac{S(t)}{4}}_{\text{absorbed solar}} - \underbrace{\sigma T^4}_{\text{IR to space}} + \underbrace{F_{\mathrm{GHG}}(C_a)}_{\text{GHG}} + \underbrace{F_{\mathrm{volc}}(t)}_{\text{volcanic}} + \underbrace{F_{\mathrm{aer}}(t, B)}_{\text{aerosol/biogenic}} . \tag{C1}$$

(With $E \approx C_T T$ one gets the usual $C_T\, dT/dt = N(t)$.)

Atmospheric CO_2 Dynamics

$$\frac{dC_a}{dt} = \underbrace{E_{\mathrm{anth}}(t) + E_{\mathrm{volc}}(t)}_{\text{direct sources}} + \underbrace{R_{\mathrm{bio}}(B, T)}_{\text{biosphere resp.}} + \underbrace{R_{\mathrm{ocean}}(T)}_{\text{ocean outgassing}} - \underbrace{GPP(B, T, CO_2)}_{\text{biosphere uptake}} - \underbrace{U_{\mathrm{ocean}}(T, C_a}_{\text{ocean uptake}}$$

(C2)

Atmospheric CO_2 Dynamics

$$F_{\mathrm{bio}}^{\mathrm{net}}(B,T,C_a) \equiv R_{\mathrm{bio}}(B,T) - GPP(B,T,CO_2), \qquad F_{\mathrm{ocean}}(T,C_a) \equiv R_{\mathrm{ocean}}(T) - U_{\mathrm{ocean}}(T,C_a).$$

$$\frac{dC_a}{dt} = E_{\mathrm{anth}}(t) + E_{\mathrm{volc}}(t) + F_{\mathrm{bio}}^{\mathrm{net}}(B,T,C_a) + F_{\mathrm{ocean}}(T,C_a). \tag{C2'}$$

Biosphere Dynamics and Condensed Climate Form

$$\frac{dB}{dt} = GPP(B, T, CO_2) \; - \; R_{\mathrm{bio}}(B, T) \; - \; L_{\mathrm{landuse}}(t, B) \; - \; D_{\mathrm{other}}(t, B). \tag{C3}$$

Biosphere Dynamics and Condensed Climate Form

$$\frac{d}{dt}\begin{bmatrix} T \\ C_a \\ B \end{bmatrix} = \underbrace{F_{\mathrm{free}}\big(T, C_a, t;\ S, F_{\mathrm{GHG}}, F_{\mathrm{volc}}, F_{\mathrm{aer}}\big)}_{\text{free-energy / forcing DOFs}} + \underbrace{F_{\mathrm{reg}}\big(T, C_a, B;\ L_{\mathrm{landuse}}, D_{\mathrm{other}}\big)}_{\text{biosphere / regulator DOFs}} + \xi(t). \tag{C4}$$

$$\frac{d}{dt}\begin{bmatrix} T \\ C_a \\ \hat{B} \end{bmatrix} = F^{\mathrm{clim}}_{\mathrm{free}}\big(T, C_a, t;\; S, F_{\mathrm{GHG}}, F_{\mathrm{volc}}, F_{\mathrm{aer}}\big) \;+\; F^{\mathrm{clim}}_{\mathrm{reg}}\big(T, C_a;\; \hat{B}\big) \;+\; \xi(t). \tag{C5}$$

(Here $\hat{B}$ is the *collapsed regulator*: the Anthropocene biosphere with land-use & other anthropogenic impacts absorbed into its state and response surfaces.)

Robotics: Standard Motion and Free-Energy Store

$$\frac{d}{dt}\begin{bmatrix} q \\ \dot{q} \end{bmatrix} = \underbrace{f_{\mathrm{mech}}(q, \dot{q})}_{\text{passive}} + \underbrace{g(q)\,u(t)}_{\text{actuation}} + \underbrace{w(t)}_{\text{disturbances}} . \tag{R1}$$

$$\dot{\xi}(t) = h\big(\xi, q, \dot{q}, \text{sensory inputs}\big). \tag{R2}$$

Robotics: Standard Motion and Free-Energy Store

$$\frac{dE_f}{dt} = -P(q, \dot{q}, u, t), \tag{R3}$$

(E_f: stored *free energy* of the robot; power draw P depends on motion and control.)

Robotics: Policy and Condensed Dynamical Form

$$u(t) = \pi\big(\xi(t), q(t), \dot{q}(t)\big). \tag{R4}$$

Robotics: Policy and Condensed Dynamical Form

$$x_{\mathrm{robot}}(t) \equiv \begin{bmatrix} q(t) \\ \dot{q}(t) \\ \xi(t) \\ E_f(t) \end{bmatrix}, \qquad \frac{dx_{\mathrm{robot}}}{dt} = \underbrace{F^{\mathrm{robot}}_{\mathrm{free}}(x_{\mathrm{robot}})}_{\text{free-energy / passive mech.}} + \underbrace{F^{\mathrm{robot}}_{\mathrm{reg}}(x_{\mathrm{robot}};\ \pi)}_{\text{controller/policy}} + w(t). \tag{R5}$$

Climate (C5)

Robot (R5)

$$\frac{dx_{\mathrm{robot}}}{dt} = F^{\mathrm{robot}}_{\mathrm{free}}\big(x_{\mathrm{robot}}\big) + F^{\mathrm{robot}}_{\mathrm{reg}}\big(x_{\mathrm{robot}};\ \pi\big) + w(t).$$

$$\frac{d}{dt}\begin{bmatrix} T \\ C_a \\ \hat{B} \end{bmatrix} = F^{\mathrm{clim}}_{\mathrm{free}}\big(T, C_a, t;\ S, F_{\mathrm{GHG}}, F_{\mathrm{volc}}, F_{\mathrm{aer}}\big) + F^{\mathrm{clim}}_{\mathrm{reg}}\big(T, C_a;\ \hat{B}\big) + \xi(t).$$

Given we can model the free energy term,

$$\dot{x} = F_{\mathrm{free}}(x, t) + F_{\mathrm{reg}}(x;\ \cdot) + \text{noise},$$

what is knowable about the future *trajectory* and *final state*?

Our efforts to predict climate change with current understanding are EXACTLY like predicting the trajectory of a robot with unknown policy, given you know it's initial charge.
Only thing you know is the LIMIT of where the robot can reach - if it goes straight without obstacles. Hardly useful information. Let alone when the robot policy is being rewritten on the fly by an active virus!

Articulation and Experiment

Epistemic class	Knowability	Mathematical handle	Measurement
Known knowns — "map size & axial directions"	KK: articulated *and* supported	F_{free} well constrained (radiative physics, cumulative forcing; $E \approx C_T T$ when used).	Belief–Plausibility ($\mathrm{Bel} \approx \mathrm{Pl}$, $\Delta \equiv \mathrm{Pl} - \mathrm{Bel} \approx 0$). .
Known unknowns — "the landscape"	KU: articulated but unproven (M, R posited; T insufficient).	F_{reg} geometry (attractors, tipping sets, basins) only partially mapped; depends on $\hat{B}$.	$\mathrm{Bel} < \mathrm{Pl}$ and $\Delta > 0$ This means two people who agree on current KK can disagree.

Epistemic class	Knowability	Mathematical handle	Measurement
Unknown unknowns — “the destination”	**UU**: outside present articulation	Which attractor we reach & by what path is underdetermined by F_{free} alone (non-identifiable without new mechanisms).	None

Equifinality

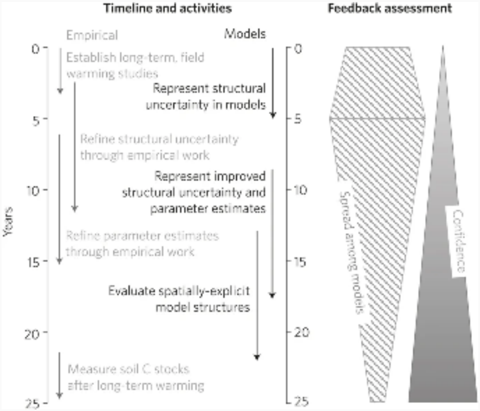
Equifinality (calibration window $[0, T_0]$):

$$\mathcal{E}_{T_0} \;\equiv\; \left\{\, \theta \in \Theta : \; F(t;\theta) = F_{\mathrm{obs}}(t), \; 0 \le t \le T_0 \right\}, \quad |\mathcal{E}_{T_0}| > 1$$

But under scenario $S: \; \exists\, \theta_1 \neq \theta_2 \in \mathcal{E}_{T_0}$ with $F_S(t;\theta_1) \neq F_S(t;\theta_2)$.

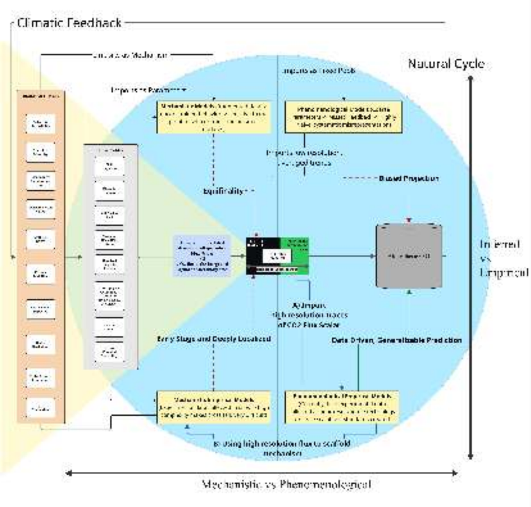
Toy case: if $F = k\,C$ and only F_{obs} is observed, any (k, C) with $k\,C = F_{\mathrm{obs}}$ is admissible.

Equifinality in Soil Carbon–Climate Feedbacks



Caption: The projection spread among soil carbon models can exceed 500 PgC even when calibrated to the same present-day stocks *(Bradford et al., 2016, Nature Climate Change).*

Conclusion of Articulation of Scientific Opportunity



Empirical/Mechanistic × Inferred/Direct landscape

Scientist - Designer, Engineer, Mathematician and

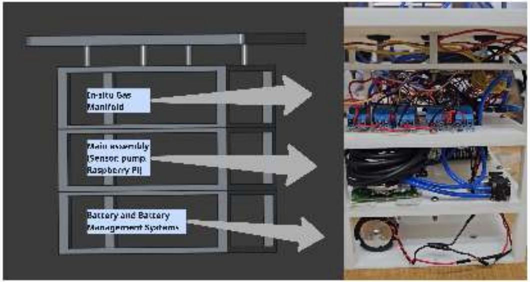
A Diagnosis of Gaps in Contemporary Methods

Criterion	Alkali Trap	Closed IRGA	Portable FTIR	DIY NDIR
Temporal resolution	Hours–days	**1–5 s**	30–60 s	**1–10 s**
Sensitivity (CO_2 flux)	$\gtrsim$0.5 μmol m^{-2} s^{-1}	$<$0.1 μmol	$<$0.2 μmol	$\sim$1 μmol
Cost / point (USD)	**$1–5**	$10–30 k	$50 k+	**$100–300**
Spatial scalability	**High (dozens)**	Low	Low	**Very high (10–100)**
Field Reliability	Traceable decay; ≈20–30 % capacity loss over 24 h	< 1 ppm day^{-1};	**« 1 ppm day^{-1} (after daily zero)**	2–5 ppm day^{-1} drift, 0.4 ppm °C^{-1} temp coeff.

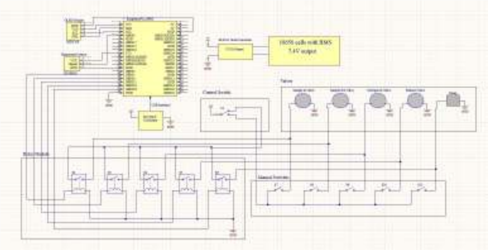
- Red cells = critical **design gaps**.

Hardware Contributions

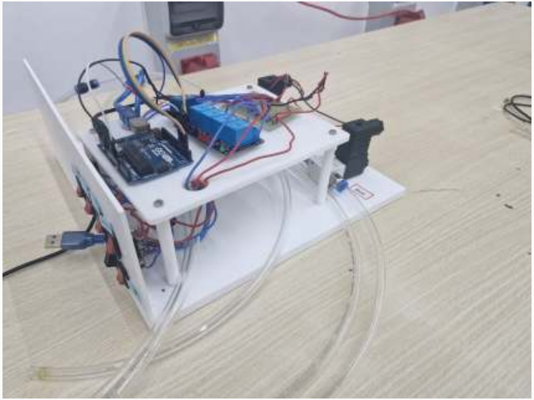
From Concept to Field-Ready Hardware



Contribution: Open-source, compute-integrated NDIR analyser (*Terrapulse*) captures sub-minute CO_2 pulses at $300 per unit.

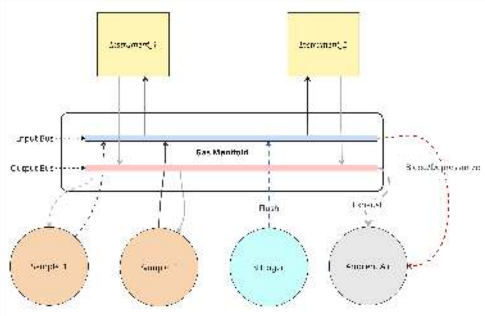


Mechanism: Beer–Lambert linearised K33-ELG cell + mini-pump; Raspberry Pi control; RTOS layer BMS



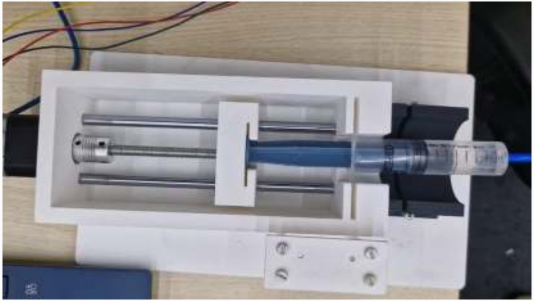
Contribution: The silent facilitator of automation and control - a universal bus for fluid multiplexing.

Run multiple samples on multiple instruments, automatically



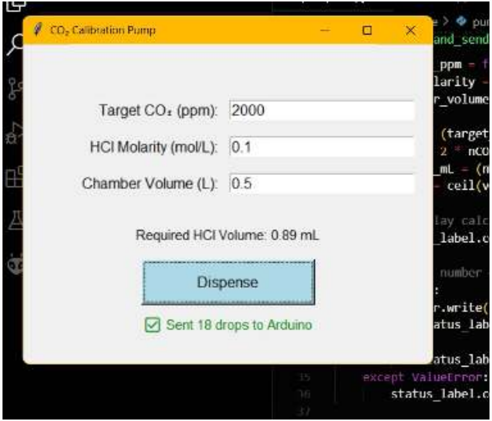
Mechanism: Dual-bus, multi-valve architecture, $\leq$50 mL dead volume => plug-flow flush in $\sim$3 s, allows indefinite extension without rise in complexity

From Concept to Field-Ready Hardware



Contribution: On-board HCl–$CaCO_3$ span generator delivers 10–2000 ppm CO_2 steps, a living standard. This is an upgraded version with step size of 10ppm, compared to earlier 100.

High complexity of chemical kinetics and fluid dynamics are masked at user end who can ping the span amount via a computer prompt



Mechanism: Stepper-driven syringe injects 0.1 M HCl into $CaCO_3$ slurry; **gold standard reference trace by itself**

Composite Hypothesis

Gap (from Slide 2)	Terrapulse	Gas Manifold	HCl–$CaCO_3$ Span Kit
High *temporal* res. & fine pulses	✓		
Low *cost* per measurement point	✓	✓	
Spatial scalability (multi-collar)		✓	✓
Field *calibration* / *drift anchor*			✓

- One open-source NDIR (**Terrapulse**) delivers sub-minute CO_2 traces at $300.
- **Gas Manifold** multiplexes collars $\Rightarrow$ 4× spatial coverage with the same sensor.
- On-board **Span Kit** injects known CO_2 steps in the field, closing the drift/accuracy loop.

Composite Hypothesis: Is Terrapulse Truly Field-Proof?

Hypothesis H_{comp}. *If a candidate instrument satisfies all six quantitative criteria below*[a]*, then its readings are causally faithful—i.e. they track true soil CO_2 dynamics within decision accuracy for pulse-resolved carbon budgeting.*

Operational realisation (three causal tests):

1. *Layer 1 – Step calibration:* can it faithfully reproduce a simple span curve?
2. *Layer 2 – Lab Birch pulse:* can it faithfully reproduce highly dynamic flux, spread over several hours?
3. *Layer 3 – Field mass closure:* can it faithfully reproduce highly dynamic flux, under varying humidity and temprature, over several days?

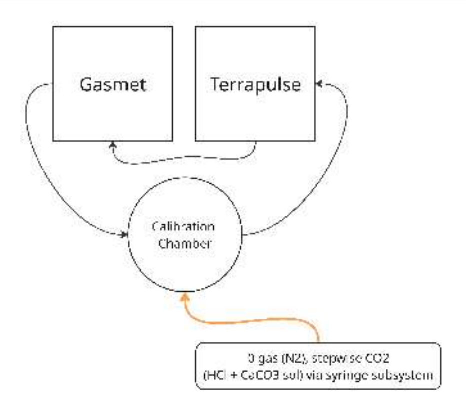
Metric	Pass band
Gain G	1.00 ± 0.02
Offset O	± 30 ppm
Drift $\lvert D \rvert$	< 0.5 ppm h^{-1}
Env. coeffs β_T, β_H	CI includes 0
Noise σ_ϵ	< 8 ppm
Flux closure ΔF	$< 5\%$

✓ = criterion satisfied at 95 % credibility. All six checks passed $\Rightarrow$ H_{comp} accepted.

[a] 95 % credibility, posterior layer 3

Experiments

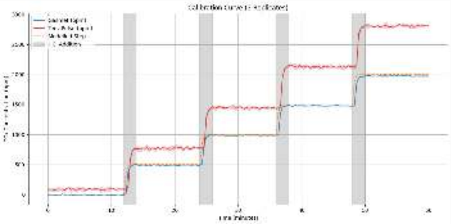
Experiment 1 — FTIR Step Calibration



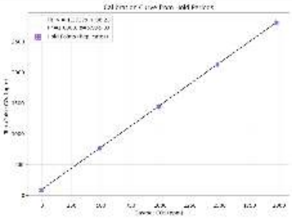
Serial flow: 0.5 L chamber → Gasmet FTIR → Terrapulse

Protocol highlights

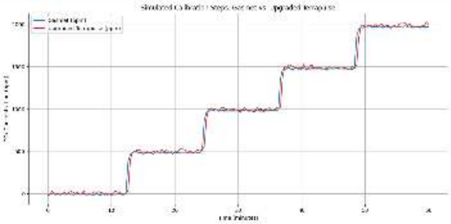
- Five CO_2 plateaus: 0 / 500 / 1000 / 1500 / 2000 ppm
- Each hold ≥ 10 min, 20 s logging interval
- First-order lag anticipated: $\tau \approx 14$ s
- Goal: derive gain G and offset O priors for Layer 2



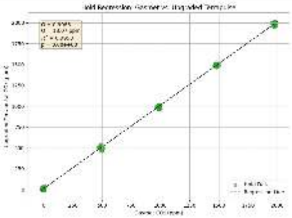
Raw span: $G = 1.37$, $O = +96$ ppm (over-response)



Hold segments (uncal.): slope 1.37, offset 96ppm

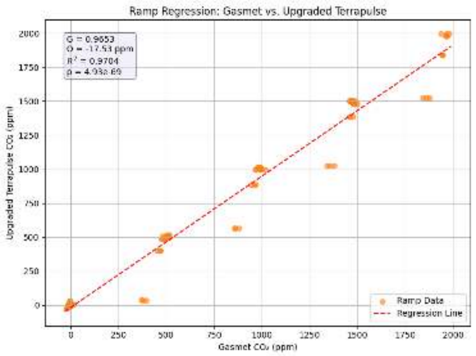


Calibrated span: $G = 0.999$, $O = +13$ ppm; $R^2 = 0.999$



Hold segments (calib.): slope 1.00 ± 0.01, offset ≈ 13ppm

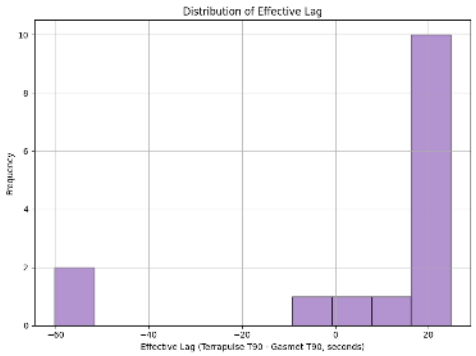
Experiment 1 — FTIR Step Calibration



Ramp vs hold: $G = 0.967$, $O = -17.5$ ppm

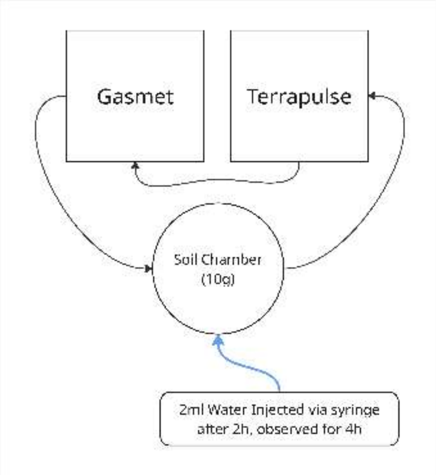
$\Rightarrow$

30ppm jump from hold with sign flip



Lag distribution peaks at 14 s; white-noise residuals

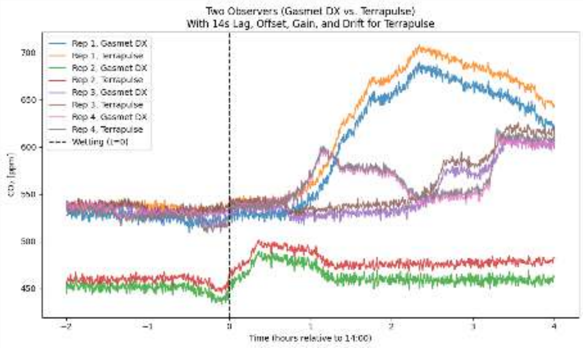
Experiment 2 — Laboratory Birch Pulse



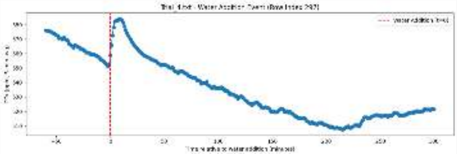
Protocol highlights

- Four biological replicates, 20 s logging for 6 h
- Controls: *abiotic* (autoclaved) and *biotic-dry* (no re-wet)
- Aim: test dynamic fidelity under a rewetting induced CO_2 surge against gold standard reference Gasmet DX.

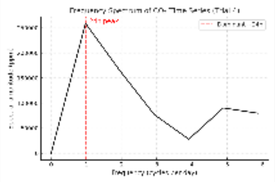
Jar microcosm: 10 g dry soil, instant re-wet at $t = 0$; parallel flow Gasmet + Terrapulse, 1 L min^{-1}



Live soil: Peak at 38 min, **12.3** μmol m^{-2}s^{-1}; area under the curve quantifies Birch magnitude.

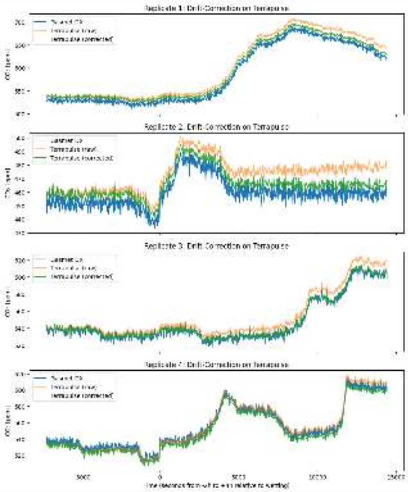


Abiotic control: Visibly Distinct Trend: Shock release and then absorb slowly; rules out mineral CO_2 release.



Biotic-dry control: No surge, All power under FFT centered around dinural rhythm

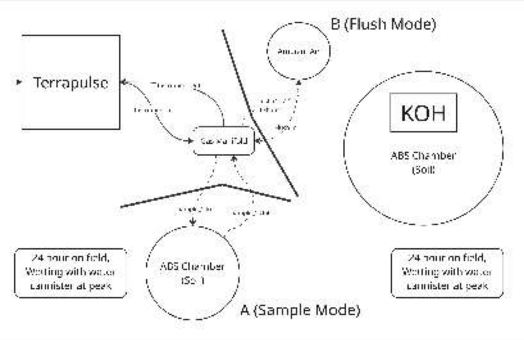
Experiment 2 — Laboratory Birch Pulse



Overlay: raw baseline (grey dashed) vs. drift-corrected trace (blue).

Baseline drift removal

- *Pre-correction fit* $G = 0.979,\ O = -11$ ppm, $\mathbf{D} = -\mathbf{0.040}$ ppm h^{-1}
- *Post-correction fit* $G = 0.995,\ O = -6$ ppm, $\mathbf{D} \approx \mathbf{0}$ ppm h^{-1} (95 % CI spans zero)



Semi-arid plot: dual collars + vent; Terrapulse in multiplexed loop, µKOH trap for 12 h integrals.

Protocol highlights

- 6-day run, chamber seal 30 min every 2 h (1 L min^{-1} flow)
- Daily 50 mL spray at 14:00; soil T, RH logged at 5 min
- KOH trap conductivity → cumulative CO_2 every 12 h
- Objective: test flux closure and env-covariate coefficients β_T, β_H

Experiment 3 — Field Mass Closure

KOH Conductance Trap

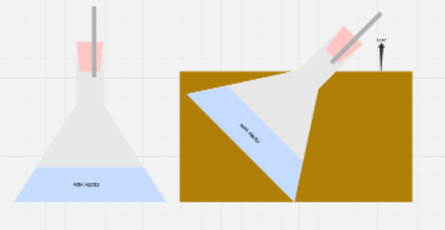
- 10 mL 0.1 M KOH, conductivity span 18 → 8 mS cm^{-1}.
- Capacity ~220 mg CO_2; random SD 0.15 mS cm^{-1}; drift < 0.01 mS cm^{-1} h^{-1}.

Decay-aware uptake model (Appendix B)

$$r_\infty = r(t_m) + \frac{r(t_m) - r_{\text{prev}}}{e^{t_m/\tau} - 1}$$

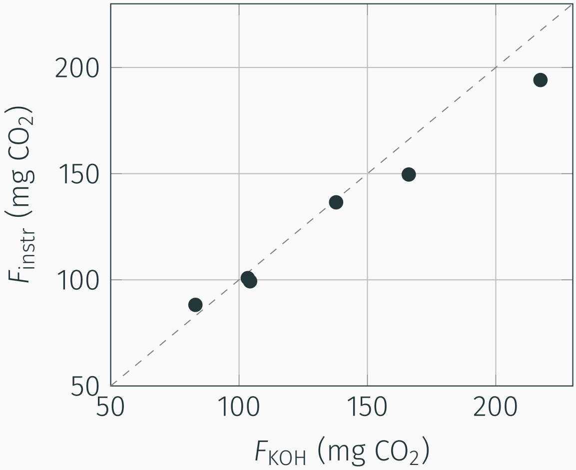
$$n_{CO_2}(t_m) = \underbrace{\kappa V}_{\text{cell factor}} (r_0 - r_\infty)\left(1 - e^{-kt_m}\right)$$

- $r(t)$: conductivity (mS cm^{-1}), r_∞: lag-corrected end-point
- τ: sensor first-order time constant (~15 s)
- k: carbonation rate constant (~0.012 s^{-1})



Field mounting: 45° conical flask

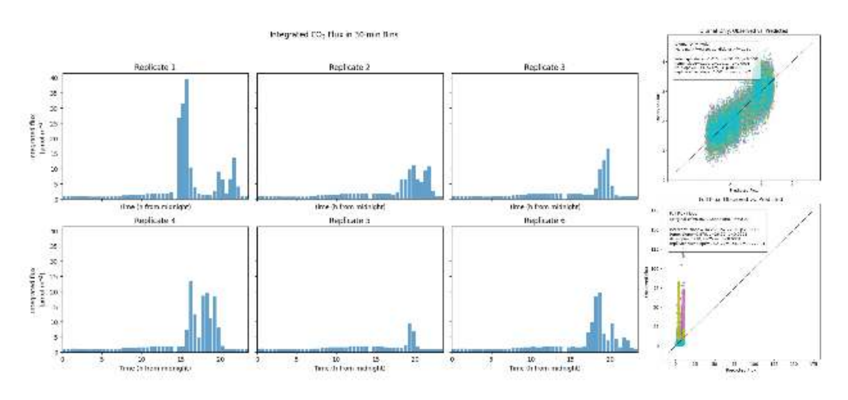
Experiment 3 — Field Mass Closure



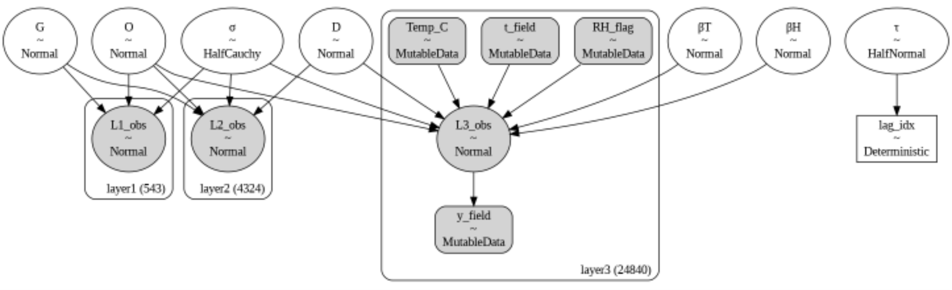
Flux-closure validation

- Six 30-min AUC points hug the 1:1 line (shaded band ±10 %).
- Mean absolute error: **3.8 %**; within the 5 % pass band.
- Confirms Terrapulse + lag correction recovers total CO_2 as accurately as µKOH anchor.

Experiment 3 — Field Mass Closure

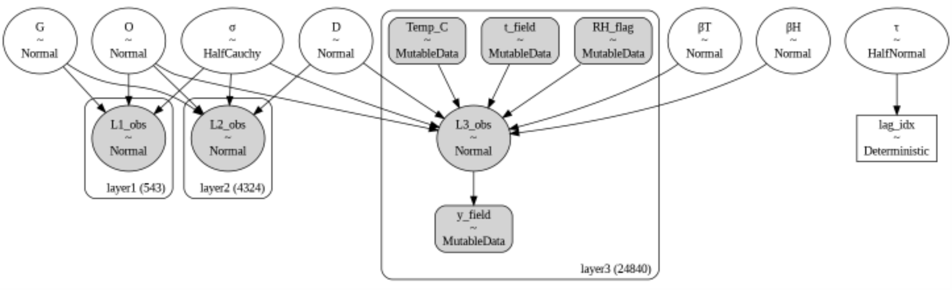


Time-resolved field trace: the 6-day record simultaneously exposes (i) discrete Birch bursts within 2 h of wetting *and* (ii) a repeating diurnal respiration crest at local 15:00. Total daily flux alone would hide both patterns—hence it is not an informative prior. This dual revelation closes the loop to Slide 1's thesis: high-resolution data collapses equifinality by disentangling fast pulses from temperature-driven background, validating our causal-sensor concept in situ.



Why chain three layers?

Instead of asking a *single* calibration to carry the entire burden of inference, we propagate information *forward*: each experiment updates G, O, τ, D, β_T, β_H, then passes its posterior as the prior for the next layer. This makes the inference *sensitive* to design choices all along the pipeline — electronic scale, dynamic pulse, and environmental drift — yet prevents any single experiment from dominating the final belief.



Priors (Layer-0)

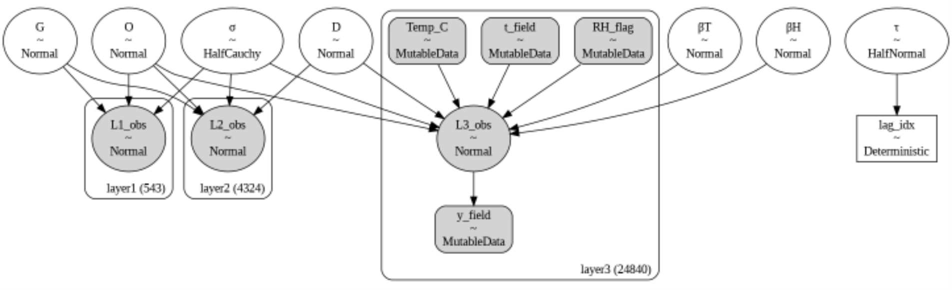
$$\theta = (G, O, \tau, D, \beta_T, \beta_H, \sigma)$$

$$G \sim \mathcal{N}(1, 0.05) \qquad O \sim \mathcal{N}(0, 50)$$

$$\tau \sim \text{HalfNormal}(5) + 10 \qquad D \sim \mathcal{N}(0, 0.10)$$

$$\beta_T \sim \mathcal{N}(0, 0.05) \qquad \beta_H \sim \mathcal{N}(0, 0.5)$$

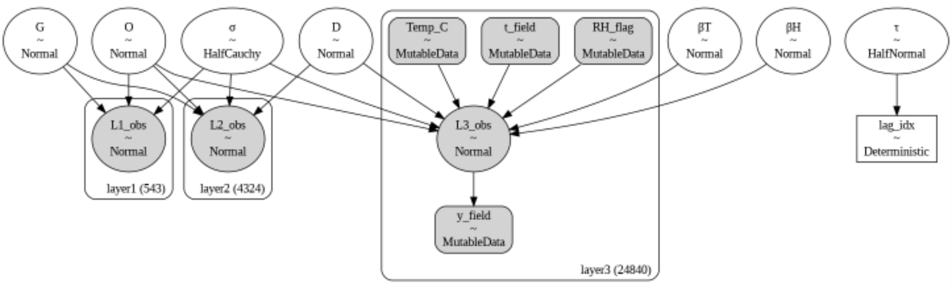
$$\sigma \sim \text{HalfCauchy}(5)$$



Three independent likelihoods

$$
\begin{aligned}
y_1 &\sim \mathcal{N}(G\,C_1 + O,\ \sigma^2) && \text{(FTIR steps)}\\
y_2 &\sim \mathcal{N}(G\,C_2(t-\tau) + O + D\,t,\ \sigma^2) && \text{(lab Birch pulse)}\\
y_3 &\sim \mathcal{N}(G\,[y_3 - f(t,T,H)] + f(t,T,H),\ \sigma^2) && \text{(field drift)}\\
F_{\text{KOH}} &\sim \mathcal{N}(F_{\text{instr}}, \sigma^2_{\text{trap}})
\end{aligned}
$$

where $f(t,T,H) = O + D\,t + \beta_T \Delta T + \beta_H H.$



Posterior update

$$p(\theta \mid y_1, y_2, y_3, F_{\mathrm{KOH}}) \;\propto\; p(y_3 \mid \theta)\, p(y_2 \mid \theta)\, p(y_1 \mid \theta)\, p(F_{\mathrm{KOH}} \mid \theta)\, p(\theta)$$

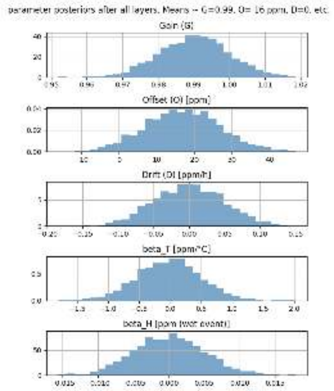
$$\theta^{(k+1)}_{\text{prior}} \;\leftarrow\; \theta^{(k)}_{\text{posterior}} \qquad (k = 1 \to 2 \to 3)$$

Each layer sharpens the credible region; final posteriors on $(G, O, \tau, D, \beta_T, \beta_H)$ are what you will see in the results slide.

Final Posterior Results

Table 1: Posterior summary after Layer 3 (mean $\pm$ 95 % CI).

Parameter	Description	Posterior
G	Gain (sensitivity)	0.993 ± 0.005
O	Offset (baseline)	16 ± 10 ppm
D	Drift rate	0.002 ± 0.020 ppm h^{-1}
β_T	Temp. bias coeff.	-0.01 ± 0.05
β_H	Humidity coeff.	0.00 ± 0.001
σ_ε	Obs. noise	~ 8 ppm



Joint posterior densities for G, O, D and others

Full convergence diagnostics and joint chains in the CO_2mmunity repo (supplementary/posteriors).

Offset Bugs → Noting Limitations

Offset phenomenon	Design-level diagnosis (double-click)
$+16$ ppm anchor jump (FTIR → KOH in Layer 3)	• *Reference change*: from pointwise FTIR trace $C_{\mathrm{FTIR}}(t)$ to scalar mass-balance n_{KOH}. • *Baseline leakage*: any small O integrates linearly in $\int_0^T (G\,C + O)\,dt$. • *No temporal diagnostics*: KOH was sampled only at T, hiding where the area mismatch originated.
$+13 \leftrightarrow -17$ ppm (ramp vs hold in Layer 1)	• *Derivative sensitivity*: ramp segments rise $\dot{C} \approx 200$ ppm min^{-1}; 15 s lag shifts reading forward. • *Plug-flow dispersion*: tubing + cell volume generate an exponential tail not captured by a pure delay. • *Equal weighting*: same σ for ramp & hold → biased slope/intercept when both pools are regressed together.

Offset Bugs → Noting Limitations

Roadmap – how we are closing both gaps

- **μKOH-20 continuous logger** • Conductivity every 60 s → time-series anchor. • Syncs via MQTT to the same Pi. • BOM $18; prototype running.
- **Two-parameter lag kernel** • Pure delay τ plus exponential tail λ^{-1}. • Ramps re-fit to G=0.999, O=+1 ppm.
- **Precision weighting option** • Keep simple kernel; assign $\sigma_{\text{ramp}} > \sigma_{\text{hold}}$. • Posterior then dominated by plateau where lag artefact is negligible.

Net result: anchor jump disappears, lag artefact neutralised, leaving a single, reference-invariant offset estimate.

Thank You

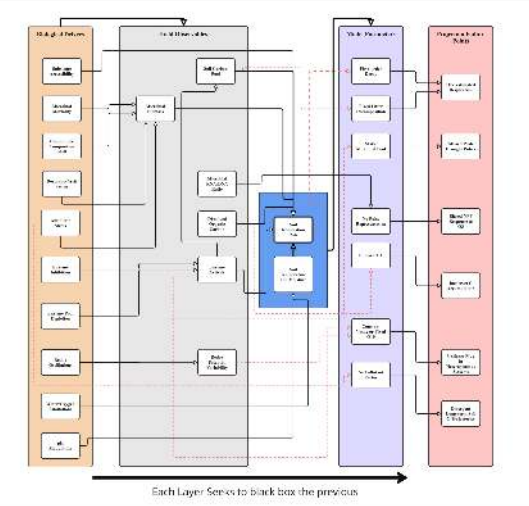
Questions?

(Appendices and full bibliography available upon request.)

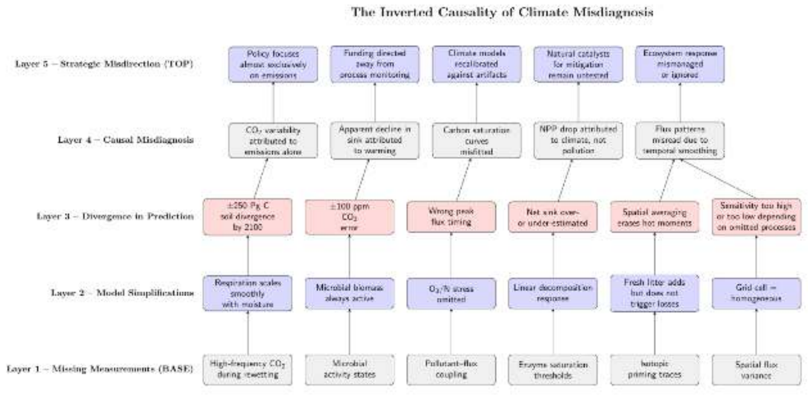
This is dedicated to my sister **Aarushi Gupta** who passed away March, 2025

Science to Policy

An example of a biospheric climate model



How bad science derails good politicians



Most of climate strategy is built atop assumptions made in place of measurements.
This diagram shows how one unmeasured variable becomes belief, projection, and policy.
This thesis inverts that chain.

The Climate Market

William Nordhaus, Nobel 2018 treat emissions as a priced externality inside growth (DICE models).

Control: mitigation μ_t (and savings s_t).
Loop: Output $\rightarrow$ Emissions $\rightarrow$ Temperature $\rightarrow$ Damages $\rightarrow$ Output.

Result: μ_t is the price of absense of CO_2 $\Rightarrow$ tradable units called carbon credits.

$$\max_{\{\mu_t, s_t\}} \sum_{t=0}^{\infty} \beta^t \, U(C_t)$$

$$Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}, \quad C_t = (1 - s_t) Y_t - \underbrace{\theta_t \mu_t^{\gamma} Y_t}_{\text{Abate}_t(\mu_t)}$$

$$E_t = (1 - \mu_t)\, \phi_t \, Y_t$$

$$(C_{t+1}, T_{t+1}) = G(C_t, T_t, E_t)$$

$$Y_t^{\mathrm{net}} = (1 - D(T_t))\, Y_t, \quad K_{t+1} = (1 - \delta) K_t + s_t Y_t^{\mathrm{net}}$$

Three Players, One Trading Surface (Interests Only)

Actor	Objective Interests (internal)	Market Handles (external)
Nations	GDP, jobs, tax intake, trade leverage, cheap capital	Carbon taxes/ETS revenue; standards; export access; climate finance inflow; nature-as-asset
Corporations	Profit, margins, cost of capital, tender eligibility	Cost pass-through, abatement/arbitrage, credit origination/purchase, subsidy capture, green premium
Banks	Risk-adjusted return, balance sheet growth, fee pools, regulatory goodwill	Green/transition loans & bonds, project finance, carbon trading/hedging, securitization

Intersection = The Climate Market

$$\mathcal{M} = \mathcal{I}_{\text{Nations}} \cap \mathcal{I}_{\text{Corporations}} \cap \mathcal{I}_{\text{Banks}}$$

Intersection = The Climate Market

- *Govt-Corp* $\Rightarrow$ **compliance** (tax / ETS).
- *Corp-Nation* $\Rightarrow$ **voluntary** market - well paying jobs, GDP.
- *Bank-Corp, Govt* $\Rightarrow$ **climate finance** (loans/bonds/investment).

Closed loop: private and public spending reallocated into these tradable objects, serving each actor's core interests.

Climate Market is, Objectively, Important

Metric (global)	Latest indicative size
Climate finance flows (annual, 2021–2022)	$\sim$\$1.3 trillion/yr
Traded value of **compliance** carbon markets (2023)	$\sim$\$0.95 trillion
Government revenue from carbon pricing (2023)	$\sim$\$104 billion
Voluntary carbon market value (2023)	$\sim$\$0.72 billion
Labelled **green bonds** issuance (2024)	$\geq$\$1.0 trillion
Private share $\approx$ 49% of total climate finance (2021–2022)	$\approx$49% of flows

Reading: compliance dwarfs voluntary; finance is the scale engine; revenue shows fiscal relevance.

What's going on?

$$\text{Households} \xrightarrow{\text{prices / bills / taxes}} \text{Firms} \xrightarrow{\text{capex \& MRV}} \text{Assets} \xrightarrow{\text{loans/bonds}} \text{Banks/Investors}$$

$$\underbrace{\text{Incidence}}_{\text{who funds}} \approx \text{pass-through in markets} + \text{fiscal recycling choices}$$

Politics 101

An Opportunity for Politicians

Objective for this class: show how climate markets give politicians *voter engagement, industrial power, and fiscal capacity*—even if they disagree with climate sentiment or specific scientific claims.

Core idea: science supplies *language*; policy supplies *instruments*; markets supply *cash-flows*. Belief is optional. Execution is not.

Scientists $\Longrightarrow$ Politicians $\Longrightarrow$ Narratives $\Longrightarrow$ Voters $\Longrightarrow$ Mandate $\Longrightarrow$ Policy Instruments

What to internalize (as a leader):

- You convert *scientific language* into *political products*: targets, rules, investments.
- The cascade is a *feedback system*: budget cycles fund better science/MRV, which sharpens future narratives and instruments.

Climate Market and your 3 Interests (Voters, Industry, Budget)

- Electoral (voter engagement)
- Industrial (power over structure) standards and subsidies tilt the field -> you pick winners.
- Fiscal (funds for Govt.): carbon taxes/ETS/fees and concessional inflows create predictable budget lines.

These levers exist whether you love or loathe climate narratives.

Markets Run on Incentives

- **Corporations** chase margin, lower WACC, and tender eligibility.
- **Banks** chase risk-adjusted return and fee pools.
- **States** chase revenue, jobs, leverage in trade/finance.

Once units (tCO_2e) and instruments (tax/ETS/credits) exist, trading/investment continues regardless if public sentiment is anti-science.

Funding science is a political investment. If it increases your voter bank, use it. Otherwise discard it.

Case Study — U.S. Under President Trump

Right Wing stance is hostile rhetoric to climate agreements and science.

What still happened (mechanics):

- Federal **PTC/ITC** incentives for wind/solar persisted; states expanded clean-energy standards.
- **Corporate PPAs** surged; utilities procured renewables on pure cost grounds.
- **Banks** financed record clean energy capacity; tax equity markets remained active.
- **Regional markets** (e.g., RGGI) continued trading; MRV infrastructure remained.

Lesson for leaders: you can say what your base wants to hear *and* still harvest jobs, capex, and fiscal benefits from an active climate market.

Voter 101

$$\textbf{Social ROI} = \frac{\Delta\text{Risk Reduction}}{\text{Total Cost to Households}}$$

$$S_{\text{voter}} = \text{Benefits}_{\text{visible}} - \text{Costs}_{\text{salient}} + \text{Narrative Bonus}$$

Real world vs Moral Story

Your future AND even your present is at stake. The climate market is for the benefit of people and paid for by the people. But it's not owned by people. Public interest in climate science will decide who benefits.

Can we fix this?

Participating in climate action is not about social responsibility points.

It is about deciding *how* larger forces control your life - climate, the biosphere, markets, and politics.

If you are a builder - engineers or entrepreneur - lock in.

You can hold the owners of these larger than life systems (states, firms, financiers) accountable in three words - **Measurement, Reporting, Verification**.

MRV GUARDS SOCIAL ROI

Measurement creates science that is rigorous enough to deserve public trust.

Reporting forces policy to be traceable back to mechanisms.

Verification keeps corporates and projects accountable to what was actually promised.

Build MRV and everyone else — governments, corporates, banks — is forced to play on your map

Lab Session

What is a Compliance Market?

A compliance market is a government-regulated carbon market where industries are legally required to measure, report, and reduce their greenhouse gas emissions under mandatory.

Mandatory Participation

Companies emitting greenhouse gases must participate in regulated monitoring and reporting systems.

Carbon Credit Requirement

Each organization must hold carbon credits equivalent to their total emissions output.

Compliance or Penalty

Non-compliance results in mandatory credit purchases or substantial financial penalties.

The Compliance Principle

> "You emit → you must show a carbon credit for each
>
> tonne → or you pay."

If the company does not have enough credits to cover its emissions, it must **buy additional credits** or **pay government-imposed penalties**.

Why This Principle Exists

- Ensures **full accountability** for emissions
- Creates a **financial cost for pollution**
- Encourages industries to **reduce emissions** instead of simply paying fines
- Guarantees that every tonne of CO_2 is **measured, verified, and offset**
- Makes the carbon market **credible and enforceable**

How It Works (Step-by-Step)

Company emits CO_2

-It must **surrender one carbon credit per tonne**
-If it does not have enough credits:

-Buy credits from the market **OR**

-Pay a penalty to the regulator

Government ensures all emissions are **reported, verified, and settled**

Why It Matters

The compliance principle transforms emissions from being an externality into a **financial obligation**.
This shifts industry behavior by:

- Driving energy efficiency
- Encouraging low-carbon technologies
- Increasing demand for verified credits
- Supporting national climate targets

Compliance Market Credit Flow

Carbon Credit Trading Scheme (CCTS), India

1. **Baseline**
   1,000,000 tCO_2
2. **Actual Emissions**
   880,000 tCO_2
3. **Reduction**
   120,000 tCO_2
4. **Regulatory Factor**
   0.95 applied
5. **Issued Credits**
   114,000 CCCs

Verification Authority

- Bureau of Energy Efficiency (BEE)
- Accredited Third-Party Verifiers
- National Steering Committee (NSC)

Financial Impact

Credit Value: \$6.77 – \$22.55 per CCC

Revenue Range: \$0.77M – 2.57M

Example:

Cement Plant Under India's CCTS Framework

01

Baseline is Set

Government or approved auditors determine historical emissions before improvements. **Example:**

1,000,000 tCO_2/year becomes the official reference point.

02

Actual Emissions are Measured

After process upgrades or efficiency improvements, current emissions are tracked. **Plant emits 880,000 tCO_2/year**, verified using official MRV systems (Monitoring, Reporting, Verification).

Credit Calculation & Issuance

1

Reduction is Calculated

Reduction = Baseline – Actual

$$\Delta E = 1,000,000 - 880,000 = 120,000 \text{ tCO}_2$$

This represents the verified emissions reduction eligible for credit claims.

2

Conservativeness Factor Applied

Regulators apply an uncertainty factor (typically 0.90–0.95) to prevent overestimation.

$$\text{Final Credits} = 120,000 \times 0.95 = 114,000 \text{ CCCs}$$

3

Credits are Issued

Government issues official **Carbon Credit Certificates (CCCs)** under CCTS. Each CCC represents 1 tonne CO_2 reduced, recorded in the national carbon registry.

Credit Utilization & Revenue Generation

Credit Trading Options

- **Internal Compliance**

Company retains credits for future regulatory obligations

- **Direct Sales**

Credits sold to companies unable to meet emission targets

- **Exchange Trading**

Traded on India's official compliance carbon exchange platform

Global Compliance Markets Landscape

EU ETS

World's largest mature cap-and-trade system with tightening emission caps and CBAM implementation from 2026

China ETS

Covers over 4 $GtCO_2$ with gradual sector expansion, focusing on power generation initially

California Cap-and-Trade

Stable market combining offset mechanisms with regular allowance auctions

India CCTS

Fastest-growing emerging compliance system with phased sector integration through 2035

Compliance Markets in Action: Global Examples

Real-world case studies demonstrate how mandatory carbon trading drives emission reductions and industrial transformation across sectors and geographies.

EU ETS: German Steel

Steel plant with 1M tonne cap invests in hydrogen steelmaking.
Emitting 900K earns 100K allowances to sell; exceeding cp requires purchasing credits

California: Oil Refinery

Refineries exceeding caps face penalties, prompting investment in carbon capture, cleaner fuels, and renewable power procurement.

China ETS: Coal Power

Plants assigned intensity benchmarks must buy permits from efficient operators or upgrade technology, driving rapid efficiency improvements nationwide.

India CCTS: Steel Sector

Plants meeting intensity targets earn CCCs for trading;those exceeding targets must purchase credits, ensuring CBAM compliance.

More Compliance Success Stories

India CCTS: Cement Industry

Plants using alternative fuels, reducing clinker ratios, and improving kiln efficiency generate surplus CCCs, accelerating low-carbon production transitions.

UK ETS: Aviation

Airlines facing emission caps shift toward Sustainable Aviation Fuel and more efficient aircraft to stay within allowance limits.

Japan JCM: Cross-Border

Japanese firms fund energy-efficient projects in Indonesia and Vietnam, earning credits for domestic compliance while supporting developing nations.

Quebec-California Link

Linked markets create larger trading pools with improved liquidity and reduced volatility, stabilizing carbon prices across borders.

Building India's Carbon Market Future

Robust MRV Infrastructure

Advanced measurement networks like Terrapulse Mini provide the accuracy foundation for credible carbon accounting

Phased Market Maturity

Systematic sector integration through 2035 ensures stability while expanding coverage and liquidity

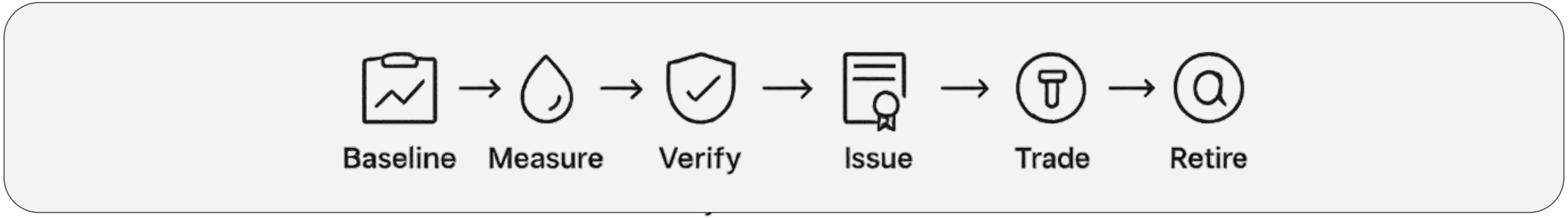
Global Integration

Article 6 readiness and CBAM alignment position India for international carbon market leadership

India’s **Carbon Credit** Trading Scheme is a breakthrough policy that ties environmental responsibility to economic gain. By **2030**, **CCTS** will evolve into a mature, globally aligned compliance market that accelerates industrial decarbonization while generating new revenue opportunities for efficiency leaders.

What is a Voluntary Market?

A **non-mandated market** where companies, sub-national entities, and individuals purchase credits to meet **net-zero claims** or **supply-chain goals**.

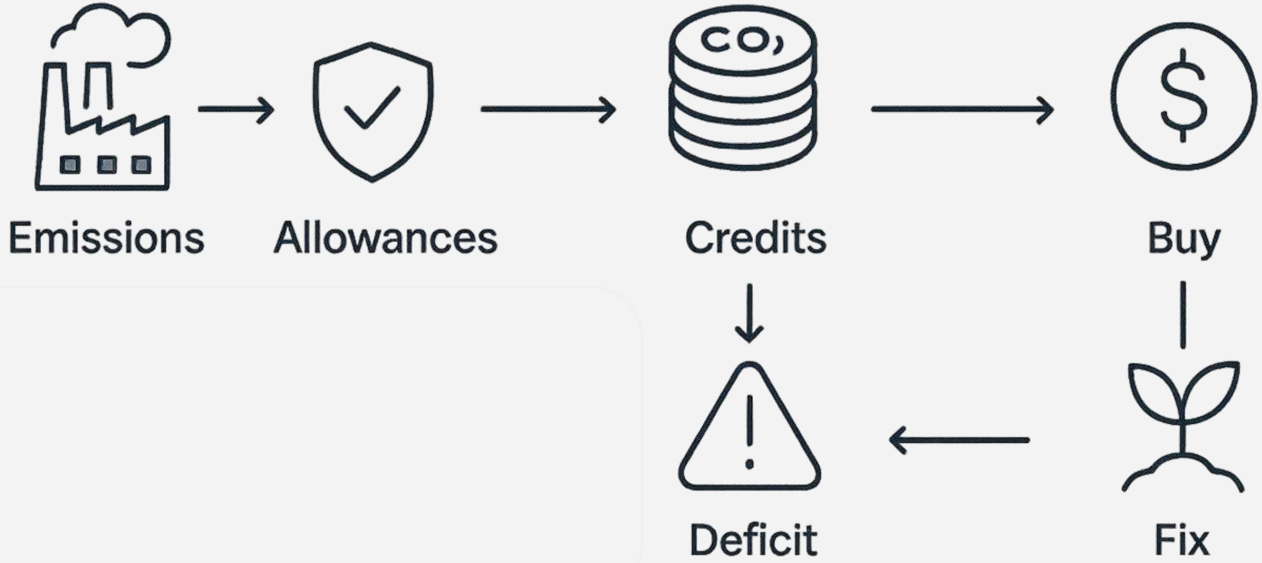


Additionallty | Leakage | Permanence | No double-counting

- Prices: Huge spread — $1 to $119 depending on method & integrity
- India: ~ 278M credits issued (2010–22) — 17% of global suoply
- Challenge: MRV bottlenecks (especially soil & AFOLU prolects)

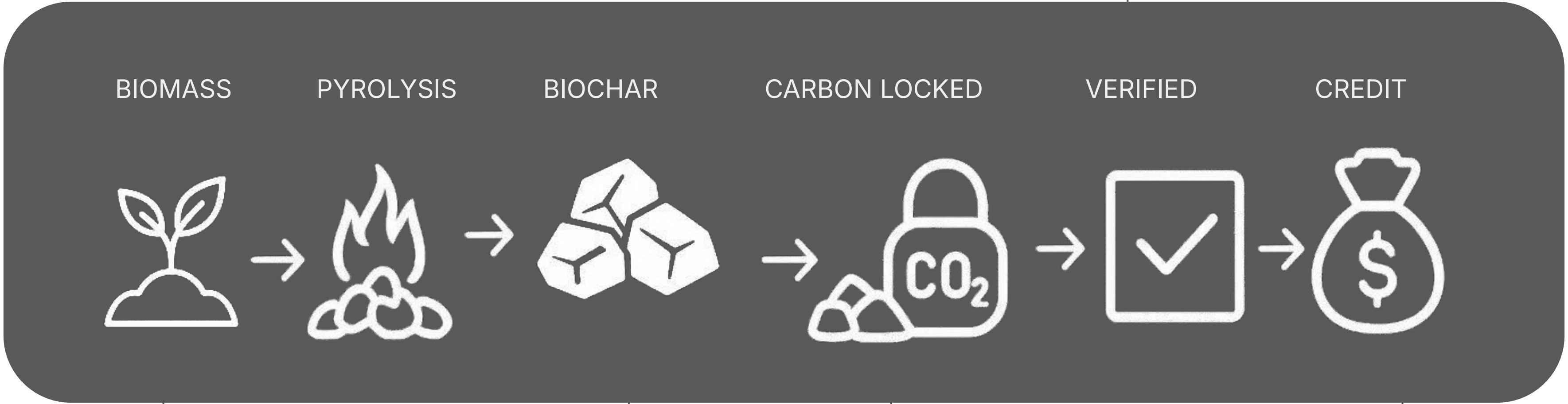
Integrity checkpoints | Leakage | Permanence |double-counting

	Compliance Markets	Voluntary Markets
Buyers	Govts, regulated entities	Companies, other actors
Primary Goal	Comply with regulation	Reduce emissions. improve supply chain
Market Size	$100+ billion (estimate)	$2 billion [room for growth]
Trajectory	Cap tightening, policy alignment	Driven by corporate demand


CO_2
Emissions
Allowances
Credits
Buy
Deficit
Fix

VCM credits = $Q_{raw} \cdot f_{\ell}\, f_{buf}\, f_{U} \approx 7{,}333.3 \times 0.90 \times 0.80 \times 0.95 \approx$ **5,013 tCO_2**

NOT ALL TONNES ARE EQUAL
leakage | permanence | uncertainty



2500 tonnes of BIOCHAR
Biochar Output - m_{bio}

80 % of CARBON stored
Carbon Fraction - w_C

$Q_{raw} = m_{bio} \cdot w_C \cdot 44/12 = 2{,}500 \times 0.8 \times 3.6667 \approx 7{,}333.3$ tCO

\$30 - \$120

\$150k - \$600k

“In carbon markets, **trust isn’t assumed — it’s engineered**.”

DESIGN

“Replace the part, not the product.”

Design for Disassembly

“Simple geometry → scalable fabrication.”

Design for Manufacturing (DFM)

“One man job.”

Lightweight & Portable

“Designed for the Soil not the Shelf.”

Field Deployability

Ease of Repair

“Because field repairs must take minutes, not hours.”

Ergonomic Form

“Comfortable to handle, even in harsh terrain.”

Ruggedized Housing

“Built to survive heat, dust, and monsoon.”

Tool-less Assembly

“Twist, lock, deploy — no tools required.”

Does Not absorb Moisture

Heat Resistance

	Acrylonitrile Styrene Acrylate	Acrylonitrile Butadiene Styrene	Polylactic Acid
UV Durability	HIGH	MODERATE	LOW
Moisture Uptake	LOW	MODERATE	MODERATE - HIGH
Printability in Field Shops	GOOD	GOOD	EASY BUT BRITTLE
Seal Integrity over Heat Cycles	HIGH	MODERATE	LOW
Sustainability Options	RECYCLABLE	RECYCLABLE	BIOPOLYMER - LOW OUTDOOR LIFE

ELECTRONICS

Tp-4056

- li-ion cell charging module.
- c type interface to make the device more relevant .

Voltage buck converter

- A step up module that gives 5V from 3.7v source.

Mq-135

- CO_2 detection module that gives analog signals

Lora sx1278

- Ultra-long-range spread spectrum transceiver
- Used for data transmission

3.7v li-ion cell

- Single-cell 18650-class Li-ion source
- Provides sufficient power and backup

Arduino nano

- Used as the main controller, runs all the peripherals as per requirement.
- Power efficient

Solar panel

- Used for sustainability.
- Used to charge the cell, and helps in long runs.

Raspberry Pi 3B+

Drives the complete device including data collection & logging and functioning of the device, uploads to the dashboard. Runs the Bayesian model.

K33 ELG CO2 sensor

CO2 sensor that gets the raw co2 in ppm unit

Sim7080G module

The gsm module that helps the device to upload data directly to dashboard using 4G data

5V Air pump

Pumps Air in the surrounding for readings

DS18B20 Temperature sensor

Gives temperature readings

5V Relay

Controls the air pump according to requirements

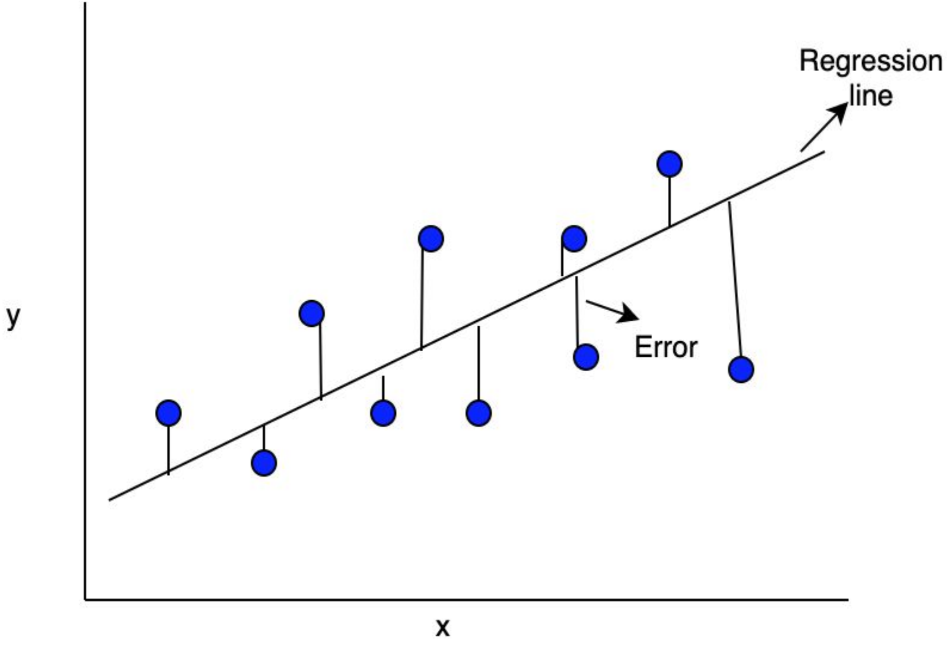
RGB Led

Used as indicator:

- RED : Error
- GREEN : Everything good
- Blinking YELLOW : Warning

STATISTICAL METHODS

Linear Regression



y = m*x + b

- y represents dependant variable
- x represents independent variable
- b is the intercept
- m is the slope of the line

Ordinary Least Squares (OLS)

$$S(m,b) = \sum_{i=1}^{n} \left(y_i - (mx_i + b)\right)^2.$$

Ordinary Least Squares (OLS)

$$S(m,b) = \sum_{i=1}^{n} \left(y_i - (mx_i + b)\right)^2.$$

$$\frac{\partial S}{\partial b} = -2\sum(y_i - mx_i - b) = 0, \qquad \frac{\partial S}{\partial m} = -2\sum x_i(y_i - mx_i - b) = 0.$$

Ordinary Least Squares (OLS)

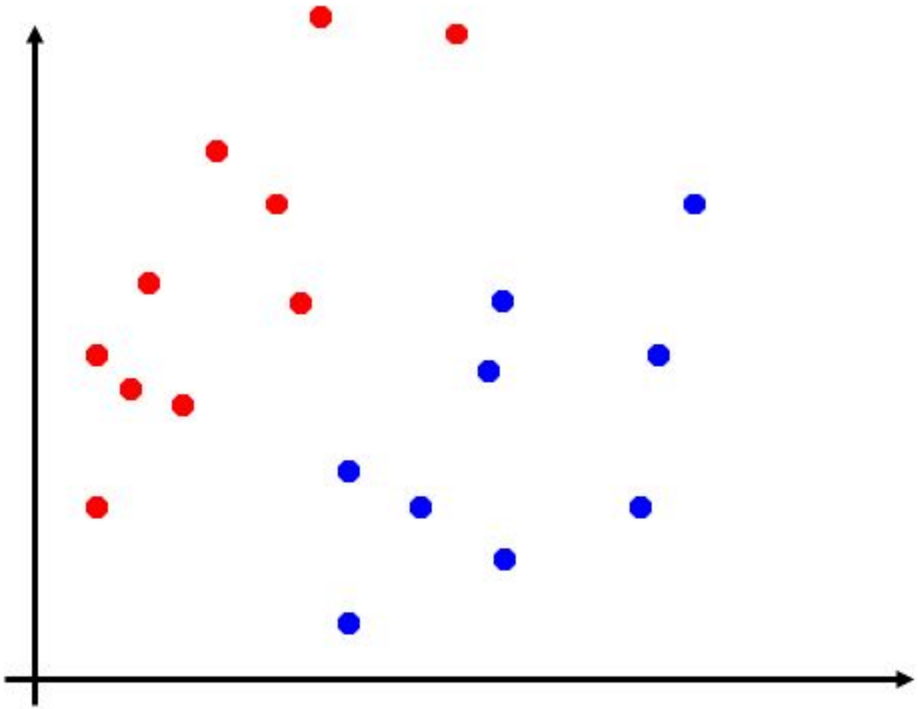
$$S(m,b) = \sum_{i=1}^{n} \left(y_i - (mx_i + b)\right)^2.$$

$$\frac{\partial S}{\partial b} = -2\sum(y_i - mx_i - b) = 0, \qquad \frac{\partial S}{\partial m} = -2\sum x_i(y_i - mx_i - b) = 0.$$

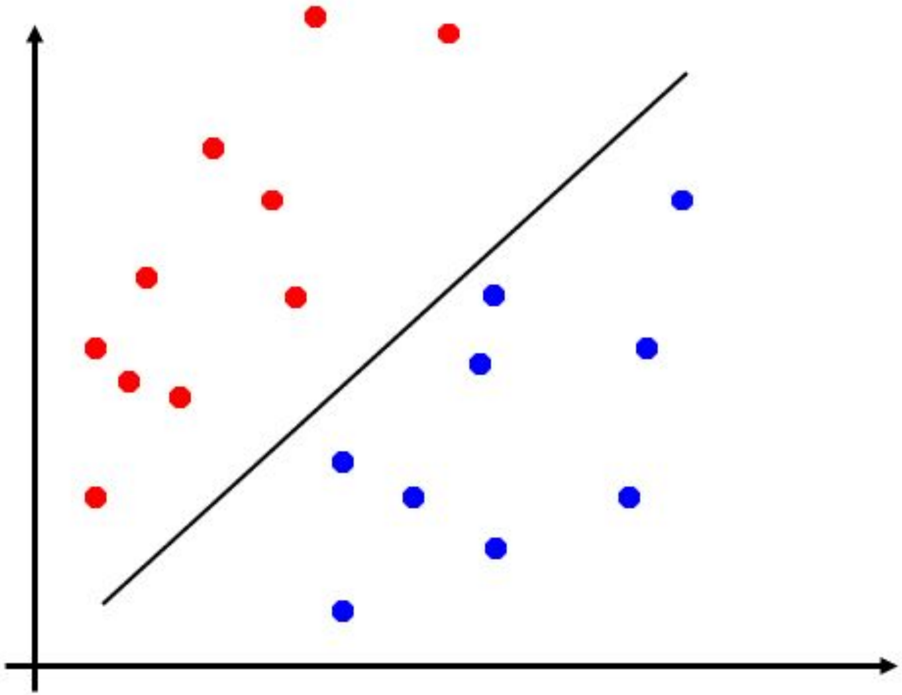
$$m = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$b = \bar{y} - m\bar{x}$$

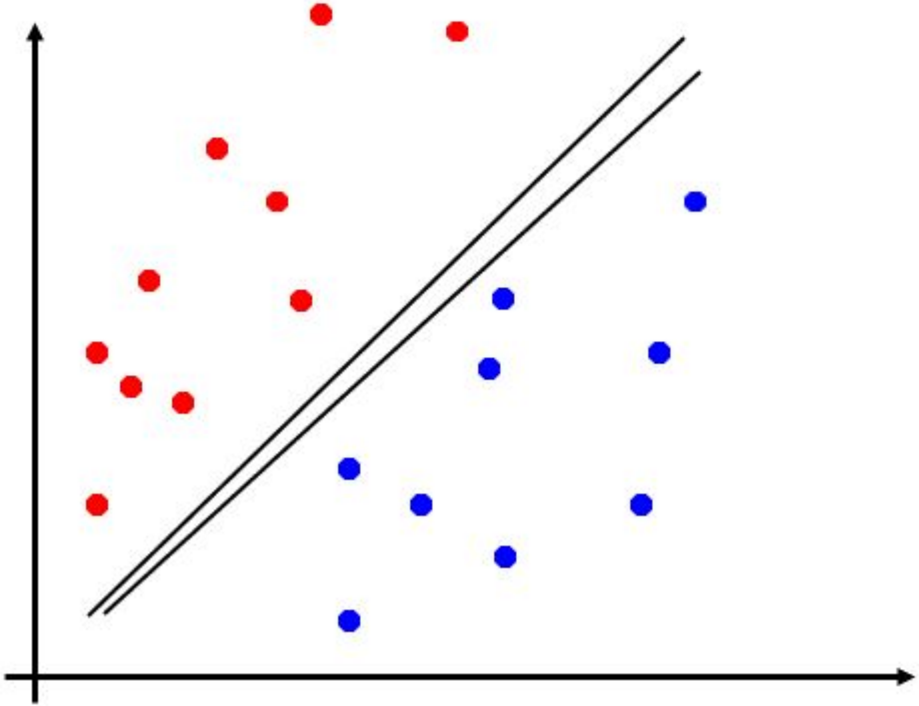
Support Vector Machines



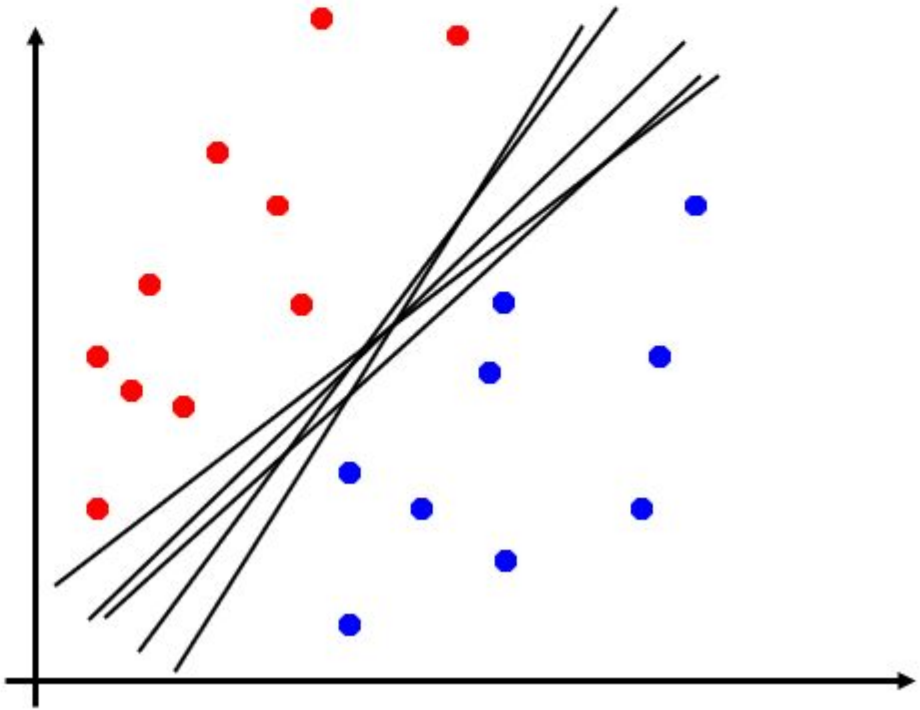
Support Vector Machines



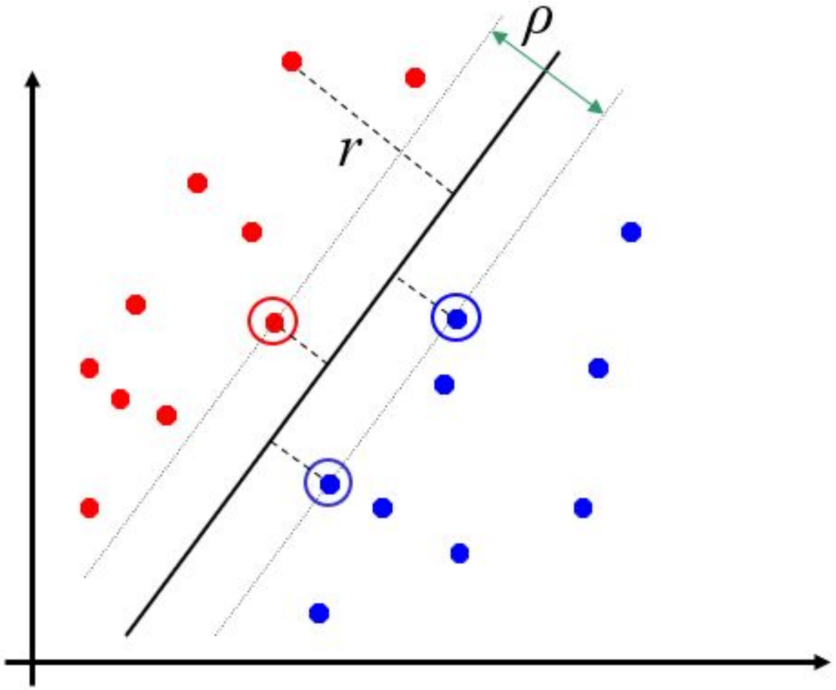
Support Vector Machines



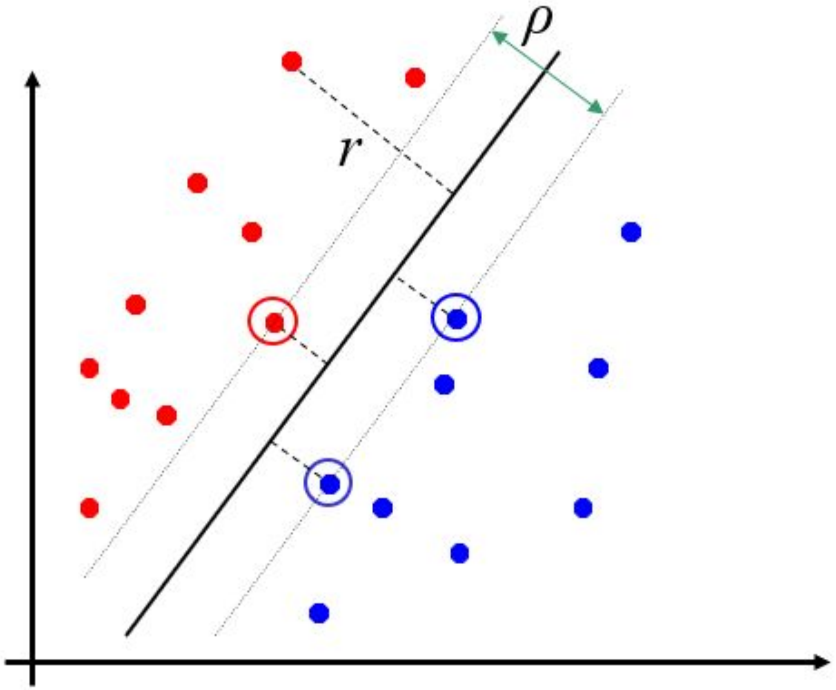
Support Vector Machines



Support Vector Machines



Support Vector Machines



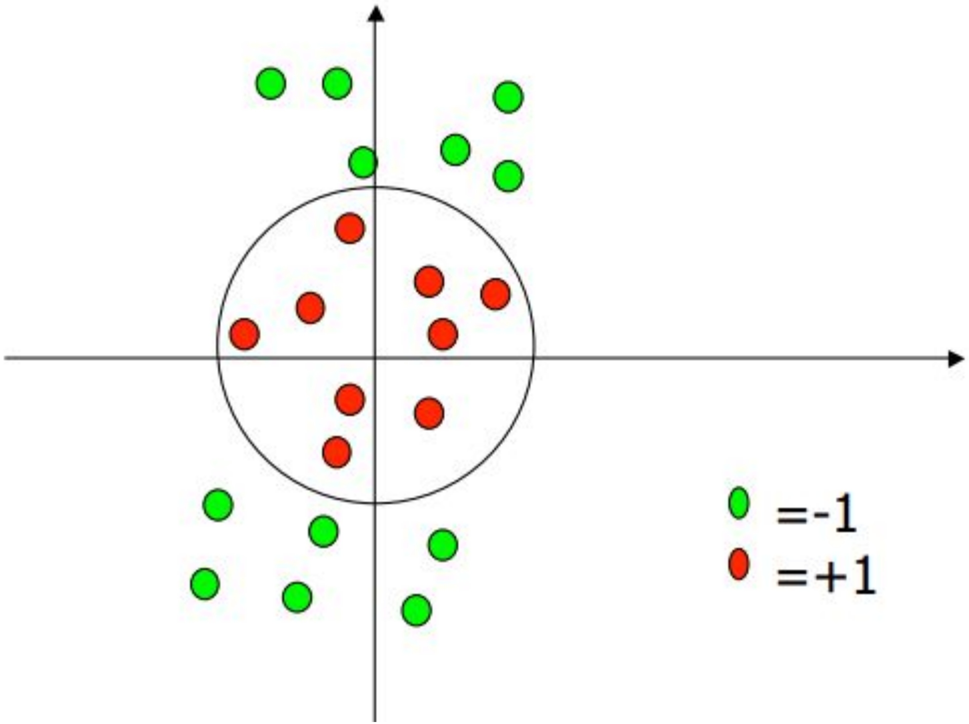
$$\mathbf{x}_i + b \leq -\rho/2 \quad \text{if } y_i = -1$$
$$\mathbf{x}_i + b \geq \rho/2 \quad \text{if } y_i = 1 \qquad \Leftrightarrow \qquad y_i(\mathbf{w}^{\mathbf{T}}\mathbf{x}_i + b) \geq \rho/2$$

$$r = \frac{y_s(\mathbf{w}^T\mathbf{x}_s + b)}{\|\mathbf{w}\|} = \frac{1}{\|\mathbf{w}\|} \qquad \rho = 2r = \frac{2}{\|\mathbf{w}\|}$$

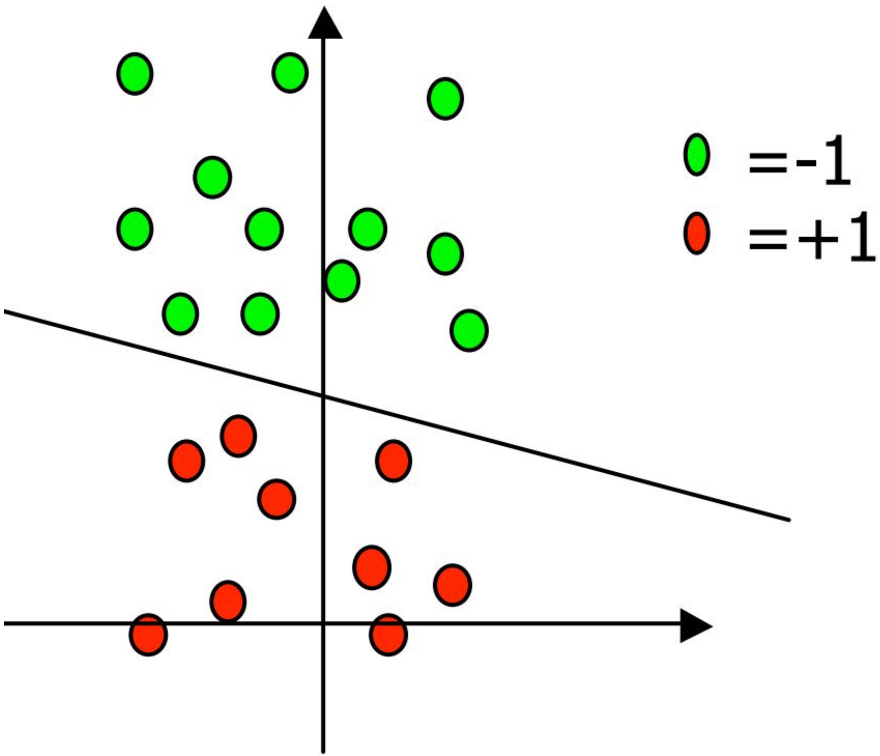
$$\mathbf{w} = \Sigma\alpha_i y_i \mathbf{x}_i \qquad b = y_k - \Sigma\alpha_i y_i \mathbf{x}_i{}^{\mathbf{T}}\mathbf{x}_k \quad \text{for any } \alpha_k > 0$$

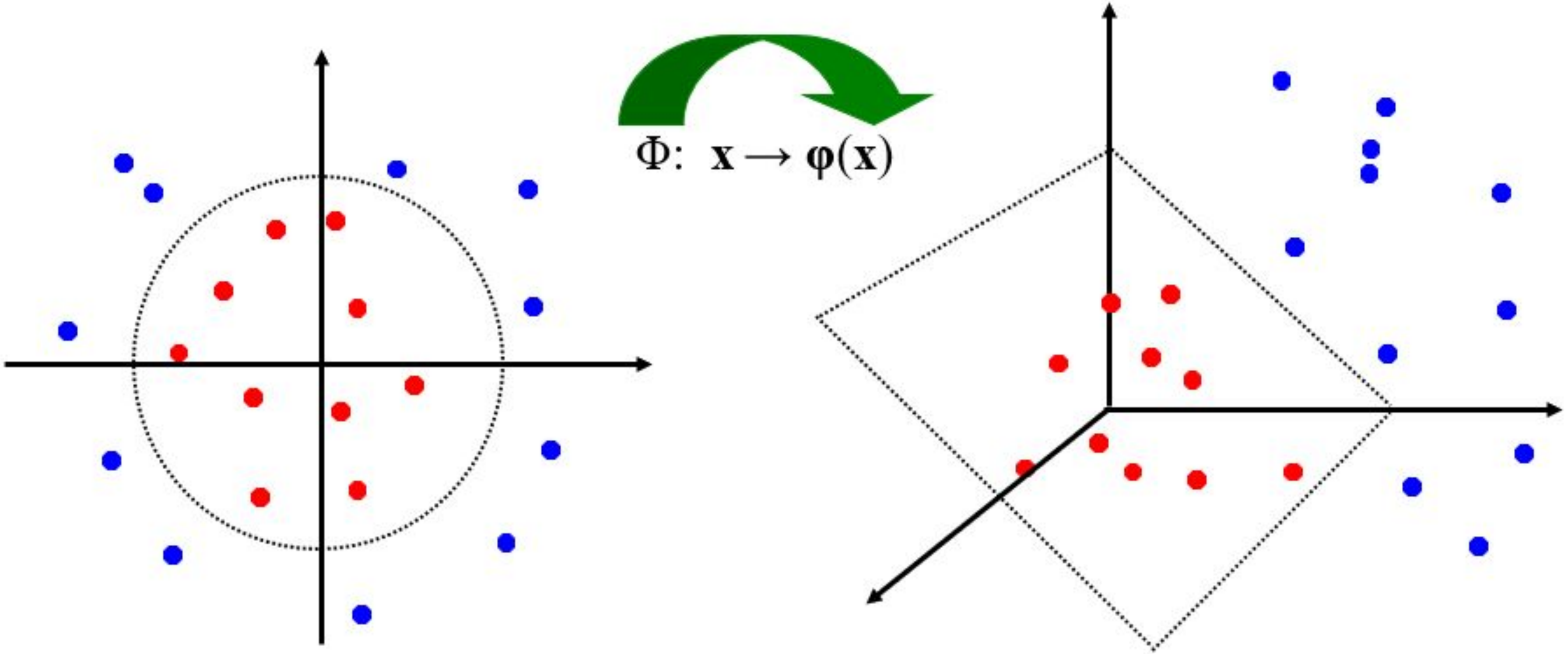
$$\boxed{f(\mathbf{x}) = \Sigma\alpha_i y_i \mathbf{x}_i{}^{\mathbf{T}}\mathbf{x} + b}$$

How can you separate these?



Polar Coordinates!




Φ: x → φ(x)

Kernel Function

$K(xi,xj) = \phi(xi) \cdot \phi(xj)$

We never compute $\phi(x)$ explicitly instead compute the kernel directly.

Kernel Function

K(xi,xj) = ϕ(xi) · ϕ(xj)

We never compute ϕ(x) explicitly instead compute the kernel directly.

Linear: $K(x,z) = x^\top z$

Polynomial: $K(x,z) = (\gamma x^\top z + r)^d$

RBF (Gaussian): $K(x,z) = \exp(-\gamma\|x - z\|^2)$

Sigmoid: $K(x,z) = \tanh(\gamma x^\top z + r)$

Bayesian Model

$$p(\theta \mid y) = \frac{p(y \mid \theta)\, p(\theta)}{p(y)}$$

P(θ) — the prior: our belief about parameter θ before seeing current data.

P(y | θ) — the likelihood: probability of observing the data if θ were true.

P(θ | y) — the posterior: our updated belief after seeing data.

P(y) — the evidence

Bayesian Model

- **Priors (Layer 0, based on references):**

$$
\begin{aligned}
G &\sim \mathcal{N}(1.0,\ \sigma_G^2) \quad \text{(gain, prior mean 1, we use } \sigma_G = 0.05 \text{ for 5\% uncertainty)},\\
O &\sim \mathcal{N}(0,\ \sigma_O^2) \quad \text{(offset, prior mean 0, we use } \sigma_O = 50 \text{ ppm)},\\
D &\sim \mathcal{N}(0,\ \sigma_D^2) \quad \text{(drift, prior 0, e.g. } \sigma_D = 0.1 \text{ ppm/hr)},\\
\beta_T &\sim \mathcal{N}(0,\ \sigma_T^2) \quad \text{(temp. effect, prior 0, small variance)},\\
\beta_H &\sim \mathcal{N}(0,\ \sigma_H^2) \quad \text{(humidity effect, prior 0, small variance)},\\
\sigma_\epsilon &\sim \text{Half-Cauchy}(0,\ \tau_\epsilon) \quad \text{(measurement noise SD, weakly-informative prior)}\ .
\end{aligned}
$$

Bayesian Model

- **Layer 1 Likelihood (Calibration data):**

$$y_i \sim \mathcal{N}(G\,C_i + O,\ \sigma_\epsilon^2), \qquad i = 1, \ldots, n_1 \ .$$

This corresponds to $p(\text{Layer1 data} \mid G, O, \sigma_\epsilon)$ as defined earlier. After Layer 1, by Bayes' rule,

$$p(G, O, \sigma_\epsilon \mid \text{Layer1}) \propto p(G)p(O)p(\sigma_\epsilon) \prod_{i=1}^{n_1} \mathcal{N}(y_i \mid GC_i + O,\ \sigma_\epsilon^2) \ .$$

Bayesian Model

- **Layer 2 Likelihood (Lab flux data):** For each time point j in the Layer 2 time-series,

$$y(t_j) \sim \mathcal{N}(\, G\,C(t_j - \tau) + O + D\,(t_j - t_{0,2})\ ,\ \sigma_\epsilon^2), \qquad j = 1, \ldots, n_2\ ,$$

$$p(G, O, D, \sigma_\epsilon \mid \text{Layer1+2}) \ \propto$$
$$p(G, O, \sigma_\epsilon \mid \text{Layer1}) \ \prod_{j=1}^{n_2} \mathcal{N}\Big(y(t_j) \mid G\,C(t_j - \tau) + O + D(t_j - t_{0,2}),\ \sigma_\epsilon^2\Big)$$

This updates D (and refines G, O) based on the continuous ramp data.

Bayesian Model

- **Layer 3 Likelihood (Field data & KOH):**
  - For each time point k in the field time-series:

$$y(t_k) \sim \mathcal{N}(\ G\,C(t_k) + O + D\,(t_k - t_{0,3}) + \beta_T[T(t_k) - T_0] + \beta_H H_{\mathrm{wet}}(t_k)\ ,\ \sigma_\epsilon^2)$$

The posterior after Layer 3 (all data) is:

$$p(G, O, D, \beta_T, \beta_H, \sigma_\epsilon \mid \text{all layers}) \propto p(G, O, D, \sigma_\epsilon \mid \text{Layer1+2})$$
$$\times \left(\prod_{k=1}^{n_3} \mathcal{N}\Big(y(t_k) \mid G\,C(t_k) + O + D\,(t_k - t_{0,3}) + \beta_T\,[T(t_k) - T_0] + \beta_H\,H_{\mathrm{wet}}(t_k),\ \sigma_\epsilon^2 \Big) \right)$$
$$\times\,\mathcal{N}\Big(F_{\mathrm{KOH}} \mid F_{\mathrm{instr}}(G, O, D, \beta_T, \beta_H),\ \sigma_{\mathrm{KOH}}^2 \Big)\ .$$

Sensor Modelling

$$y_t = G\,C_t + O + D\,(t - t_0) + \beta_T\,\Delta T_t + \beta_H\,H_t + \epsilon_t$$

$$\theta \equiv (G, O, D, \beta_T, \beta_H), \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2), \quad \theta \sim \mathcal{N}(\mu_0, \Sigma_0)$$

Sensor Modelling

$$y_t \;=\; G\,C_t + O + D\,(t - t_0) + \beta_T\,\Delta T_t + \beta_H\,H_t + \epsilon_t$$

$$\theta \equiv (G, O, D, \beta_T, \beta_H), \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2), \quad \theta \sim \mathcal{N}(\mu_0, \Sigma_0)$$

$$\widehat{C}_t \;=\; \frac{y_t - \widehat{O} - \widehat{D}(t - t_0) - \widehat{\beta_T}\Delta T_t - \widehat{\beta_H} H_t}{\widehat{G}}$$

Sensor Modelling

$$y_t \;=\; G\,C_t + O + D\,(t - t_0) + \beta_T\,\Delta T_t + \beta_H\,H_t + \epsilon_t$$

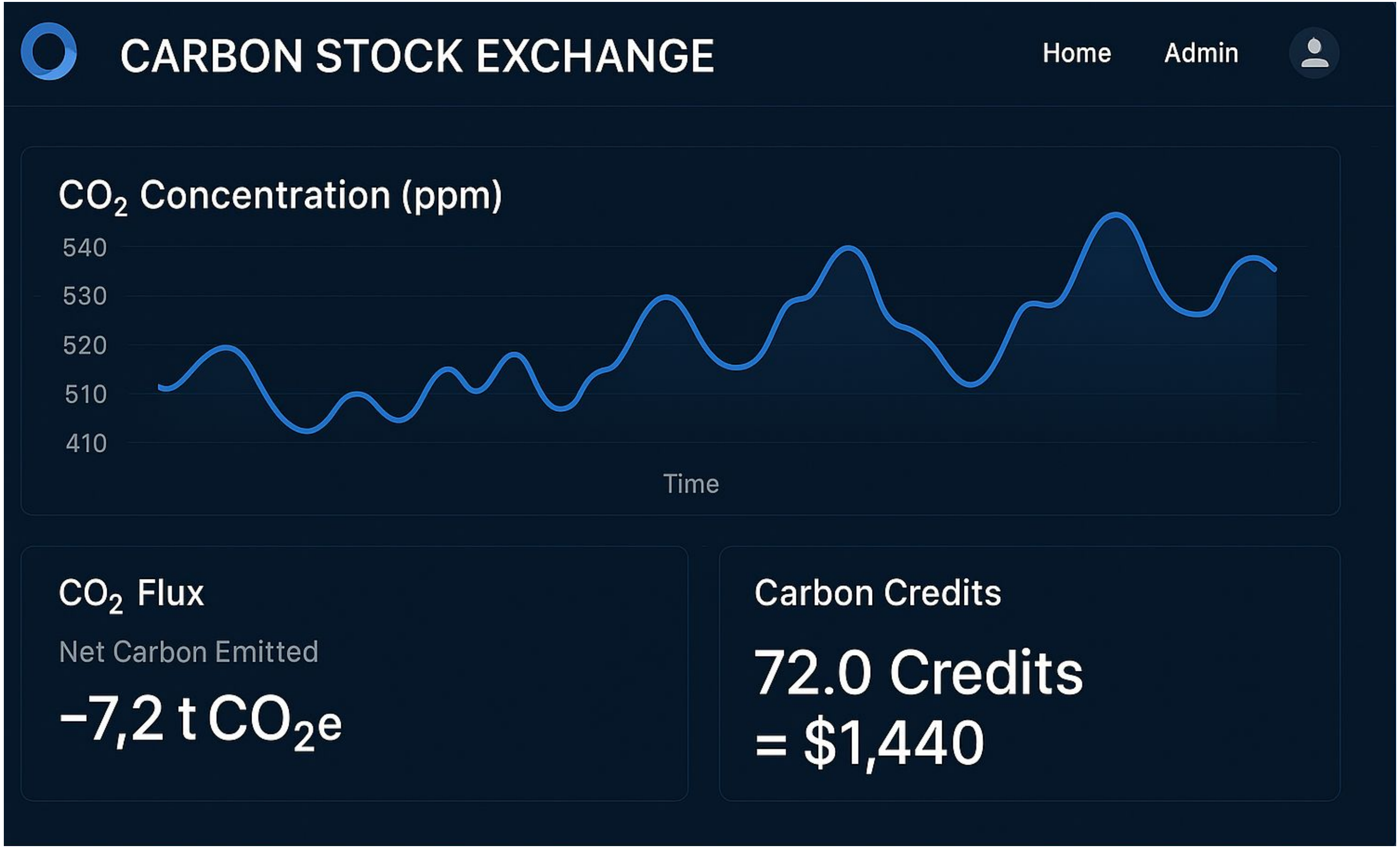
$$\theta \equiv (G, O, D, \beta_T, \beta_H), \quad \epsilon_t \sim \mathcal{N}(0, \sigma^2), \quad \theta \sim \mathcal{N}(\mu_0, \Sigma_0)$$

$$\widehat{C}_t \;=\; \frac{y_t - \widehat{O} - \widehat{D}(t - t_0) - \widehat{\beta_T}\Delta T_t - \widehat{\beta_H} H_t}{\widehat{G}}$$

$$\lim_{n\to\infty} \|\widehat{\theta} - \theta^\star\|_2 \xrightarrow{\mathbb{P}} 0 \;\Rightarrow\; \sup_{t\leq T} |\widehat{C}_t - C_t| \xrightarrow{\mathbb{P}} 0$$

Parameter / Test	**Prior (Expectation)**	**Posterior Layer 1**	**Posterior Layer 2**	**Posterior Layer 3 (Final)**
Gain G (sensitivity)	1.00 ± 0.05	0.99 ± 0.01	0.990 ± 0.008	0.990 ± 0.008
Offset O (ppm)	0 ± 50	5 ± 5	5 ± 5	3 ± 5
Drift D (ppm/hr)	0 ± 0.1	– (N/A)	-0.2 ± 0.3	-0.2 ± 0.3
Temp. effect β_T (ppm/°C)	0 ± 1	–	–	-0.1 ± 0.2
Humidity effect β_H (ppm wet)	0 ± 5	–	–	-4 ± 5
Random error σ_ϵ (ppm SD)	(spec-based, e.g.	5.5	5.2	5.3

CO2MMUNUTY DASHBOARD


CARBON STOCK EXCHANGE
Home
Admin
CO_2 Concentration (ppm)
540
530
520
510
410
Time
CO_2 Flux
Net Carbon Emitted
−7,2 t CO_2e
Carbon Credits
72.0 Credits
= $1,440